Convergence Acceleration for a Three-Dimensional Euler/Navier-Stokes Zonal Approach

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Abstract

A FAST diagonal algorithm is coupled with a zonal approach to solve the three-dimensional Euler/Navier-Stokes equations. Transonic viscous solutions are obtained on a 150,000-point mesh for an NACA 0012 wing. A three-order-of-magnitude drop in the L2 norm of the residual requires approximately 500 iterations, which takes about 45 min of CPU time on a Cray-XMP. Effects on convergence rate due to increased zonal boundary overlap regions, different stretching distributions in the viscous regions, and different Δt_0 values are also explored.

Contents

The contribution of this synoptic lies in its new computational approach in which the fast-convergent Pulliam-Chaussee[1] diagonal algorithm is coupled with a zonal approach. The original algorithm coded (TNSCY3) necessitated inverting block tridiagonals during the solution procedure, whereas the diagonal algorithm (TNSCY4) inverts scalar pentadiagonals. The scalar pentadiagonals arise due to the fourth-order implicit dissipation terms of the algorithm. The convergence rate was further enhanced by the use of spatially varying time steps. Thin-layer Navier-Stokes equations are used for the computations. To initiate the zonal approach, first a coarse grid (zone 1) is generated about the configuration in question. For this case, the geometry was an NACA 0012 wing with 20-deg sweep, an aspect ratio of 3.0, a taper ratio of 1.0, and zero dihedral and twist. Figure 1 illustrates the wing grid and a chordwise slice of zones 1-3 at the symmetry plane ($y=0$). Zone 4 (not seen in Fig. 1) occupies the lower area of the wing, including the lower surface of the wing. Zone 2 (finer inviscid) and zones 3 and 4 (viscous) are generated from the base zone 1. For complete details of the algorithm implementation and the zoning procedure, see Ref. 2.

The flow conditions were $M_\infty=0.826$ and $\alpha=2.0$ deg, and a Reynolds number based on a chord of 8 million, which is a moderately difficult case involving a strong shock and mild separation. The convergence rate of the diagonal version of the code (TNSCY4) vs the nondiagonal version (TNSCY3) is shown in Fig. 2. The abscissa denotes the number of iterations for the algorithm, and the ordinate denotes the L2 norm of the residual. The time step used in the nondiagonal version was $\Delta t=0.004$. This was the largest time step possible while still maintaining stability of the code. The diagonal version used a variable time step $[\Delta t=\Delta t_0/(1.0+\mathrm{SQRT}(J))]$, where J is the Jacobian and Δt_0 is a variable parameter, with a default value of 5. The slow rate of convergence in TNSCY3 seems to occur in the outer inviscid zones. The residual in the viscous zones in the first thousand iterations drops fairly fast and then begins to flatten out. In 5000 iterations, all zones have dropped about two orders of magnitude in the L2 norm of the residual. In contrast, the convergence rate of the diagonal version drops rapidly in all of the zones. A three-order-of-magnitude drop in the L2 norm occurs in about 400-500 iterations. This convergence rate (coupled with decrease in arithmetic operation count due to the diagonal algorithm) increases the speed with which solutions are obtained by a factor of 40.

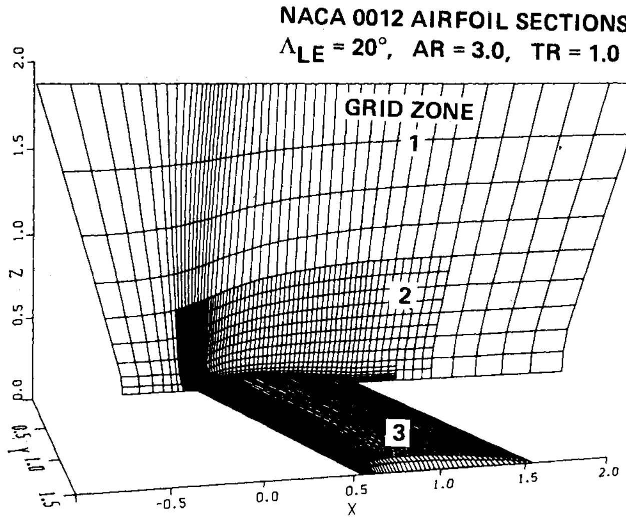

Fig. 1 Chordwise slice of zonal topology at symmetry ($y=0$) plane.

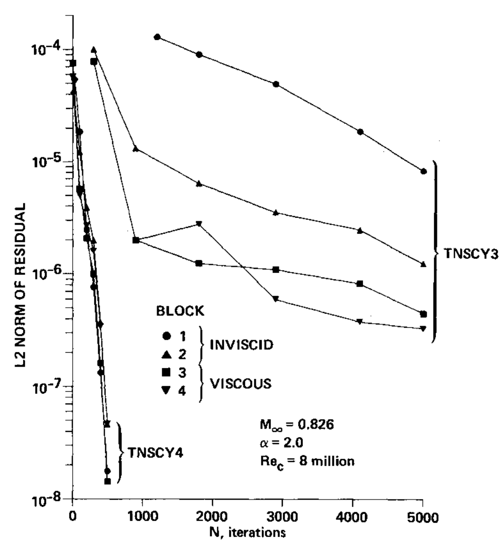

Fig. 2 Comparison of convergence rate, TNSCY3 vs TNSCY4.

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Table 1 Parametric study on convergence rates

	NB			
	1	2	3	4
NOVER = 1	>1.0	>1.0	>1.0	0.88
= 2	>1.0	>1.0	>1.0	0.92
= 3	>1.0	0.99	>1.0	0.92
KRAT2 = 2	>1.0	0.97	>1.0	0.90
= 3	>1.0	>1.0	>1.0	0.92
= 4	>1.0	>1.0	>1.0	0.99
$\Delta t_0 = 2.5$	>1.0	>1.0	>1.0	>1.0
(Default) = 5.0	1.0	1.0	1.0	1.0
= 7.5	>1.0	0.72	0.82	0.72
= 10.0	>1.0	0.61	>1.0	0.71
Optimal Δt_0/zone	0.90	0.84	0.90	0.81

The parameter that controls the amount of zone overlap is called NOVER. A value of NOVER = 1 yields a one coarse-cell overlap of zone 1 to zone 2, and a one fine-cell overlap from zone 2 to the viscous zones. The KRAT2 parameter controls the number of viscous points in the one fine-grid-cell overlap between zone 2 to zone 3 or 4 and, more importantly, changes the stretching distribution in the viscous zones. Because of the complexity of the zoner code (which develops zones 2-4 from zone 1), only the overlap regions between zones 1 and 2 in the normal direction were changed, and the overlap regions in the chordwise and spanwise directions were kept fixed at default value (NOVER = 1). In order to vary the values of NOVER, the grid had to be decreased to a 100,000-point mesh because of dimensional restrictions of the code.

With NOVER, KRAT2, and Δt_0 set at default values, the calculations for the default case were stopped at 500 iterations, since the L2 norm of the residual had dropped the required three orders of magnitude. Table 1 indicates the number of iterations (normalized by 500) required to achieve the same level of convergence as the default case for the different studies. Values greater (less) than 1, reflect a greater (lower) number of iterations required for a comparable level of convergence as the default case. The first study was to assess the convergence effect, due to NOVER varying from 1 to 3. This increases slightly the number of points in the fine grid (zone 2) that have to be calculated. It was anticipated that increasing the overlap in the zonal interfaces would tend to make the explicit boundary procedure less pronounced and possibly accelerate the convergence rate. However, comparing the values in Table 1 in each zone for the different values of NOVER showed no significant trends. In fact, zones 1-3 showed an increase in the number of iterations required. Only zone 4 showed about a 10% improvement, i.e., 50 iterations less than required in the default case.

Next, the KRAT2 parameter was varied to determine its effects on the convergence rates in the different zones. Decreasing KRAT2 decreases the number of viscous points in the fine-grid cell but increases the clustering of points in the normal direction in the viscous zones. Table 1 illustrates the effects of different values of KRAT2 for fixed NOVER = 2. KRAT2 = 3 is the usual default value. Using KRAT2 = 4 tended to slow down the convergence rate in all of the zones. However, decreasing KRAT2 to a value of 2 increased the convergence rate in zones 2 and 4. The greatest increase was about 10% in zone 4. Again, the overall trend was not very significant with the variation of KRAT2.

In an attempt to demonstrate the robustness of the new procedure, different values of Δt_0 were used. These values ranged from 2.5 to 10.0, in increments of 2.5. Table 1 shows the quite dramatic effects on the convergence rates in all of the zones. Δt_0, equal to the default value of 5, was the optimal value for zone 1. Other values tended to slow down the convergence rate for zone 1. However, $\Delta t_0 = 5$ is not necessarily the optimal Δt_0 value for the other zones. Zone 2 showed an improvement in the 10-40% range for $\Delta t_0 = 10.0$ relative to the convergence rates of $\Delta t_0 = 7.5$ and 5.0, respectively. Zone 3 showed an improvement of about 20%, and zone 4 an improvement of 30% relative to the default convergence rate. Values of Δt_0 up to 15 were also used, with the result that stability was still maintained, although the convergence rate started to degrade in all zones. This illustrates that the largest time step possible is not necessarily the optimal time step for convergence.

In Table 1, Δt_0 values of 5.0, 10.0, and 7.5 were used in zones 1,2, and the viscous zones, respectively. These results were compared with the default value of Δt_0. In the inviscid zones, the increase in convergence is about 10–16%, and in the viscous zones the increase is on the order of 10–20%. The overall increase in convergence ranged from 10 to 20%. Since this test case required about 45 min of CPU time, a 10–20% increase translates into 4–9 min of CPU savings. Keep in mind that this increase was obtained with no additional overhead cost.

Conclusions

A fast diagonal algorithm has been successfully implemented within the framework of a zonal approach. Results indicate that the new code (in obtaining a solution for a moderately difficult case) still maintains its fast convergence characteristics. Different values of NOVER (overlap parameter) in the inviscid zones show no significant effects on the convergence rates. However, increasing the clustering (decreasing KRAT2) seems to enchance the convergence rate. The use of different Δt_0 values in the different zones shows increases of 10–20% in convergence rates.

References

[1]Pulliam, T. H. and Chaussee, D. S., "A Diagonal Form of an Implicit Approximate-Factorization Algorithm," *Journal of Computational Physics,* Vol. 39, No. 2, 1981, pp. 347-363.

[2]Holst, T. L., Kaynak, U., Gundy, K. L., Thomas, S. D., and Flores, J., "Numerical Solution of Transonic Wing Flows Using an Euler/Navier-Stokes Zonal Approach," AIAA Paper 85-1640, 1985.